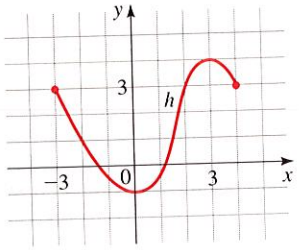
2.2 Exercises

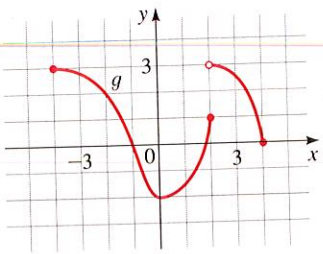
1–22 ■ Sketch the graph of the function by first making a table of values.

1. $f(x) = 2$
2. $f(x) = -3$
3. $f(x) = 2x - 4$
4. $f(x) = 6 - 3x$
5. $f(x) = -x + 3, \quad -3 \le x \le 3$
6. $f(x) = \dfrac{x - 3}{2}, \quad 0 \le x \le 5$
7. $f(x) = -x^2$
8. $f(x) = x^2 - 4$
9. $g(x) = x^3 - 8$
10. $g(x) = 4x^2 - x^4$
11. $g(x) = \sqrt{x + 4}$
12. $g(x) = \sqrt{-x}$
13. $F(x) = \dfrac{1}{x}$
14. $F(x) = \dfrac{1}{x + 4}$
15. $H(x) = |2x|$
16. $H(x) = |x + 1|$
17. $G(x) = |x| + x$
18. $G(x) = |x| - x$
19. $f(x) = |2x - 2|$
20. $f(x) = \dfrac{x}{|x|}$
21. $g(x) = \dfrac{2}{x^2}$
22. $g(x) = \dfrac{|x|}{x^2}$

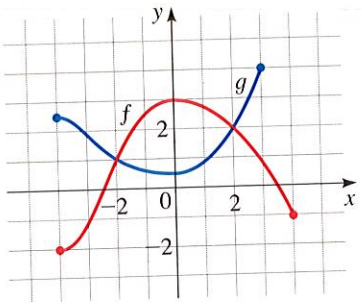
23. The graph of a function h is given.
    (a) Find $h(-2)$, $h(0)$, $h(2)$, and $h(3)$.
    (b) Find the domain and range of h.



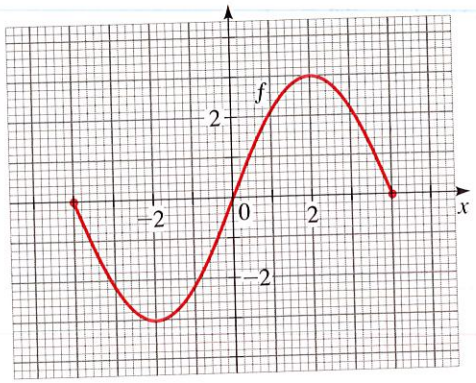
24. The graph of a function g is given.
    (a) Find $g(-4)$, $g(-2)$, $g(0)$, $g(2)$, and $g(4)$.
    (b) Find the domain and range of g.



25. Graphs of the functions f and g are given.
    (a) Which is larger, $f(0)$ or $g(0)$?
    (b) Which is larger, $f(-3)$ or $g(-3)$?
    (c) For which values of x is $f(x) = g(x)$?



26. The graph of a function f is given.
    (a) Estimate $f(0.5)$ to the nearest tenth.
    (b) Estimate $f(3)$ to the nearest tenth.
    (c) Find all the numbers x in the domain of f for which $f(x) = 1$.



27–36 ■ A function f is given.
(a) Use a graphing calculator to draw the graph of f.
(b) Find the domain and range of f from the graph.

27. $f(x) = x - 1$
28. $f(x) = 2(x + 1)$
29. $f(x) = 4$
30. $f(x) = -x^2$
31. $f(x) = 4 - x^2$
32. $f(x) = x^2 + 4$
33. $f(x) = \sqrt{16 - x^2}$
34. $f(x) = -\sqrt{25 - x^2}$
35. $f(x) = \sqrt{x - 1}$
36. $f(x) = \sqrt{x + 2}$

37–50 ■ Sketch the graph of the piecewise defined function.

37. $f(x) = \begin{cases} 0 & \text{if } x < 2 \\ 1 & \text{if } x \ge 2 \end{cases}$

38. $f(x) = \begin{cases} 1 & \text{if } x \le 1 \\ x + 1 & \text{if } x > 1 \end{cases}$

39. $f(x) = \begin{cases} 3 & \text{if } x < 2 \\ x - 1 & \text{if } x \ge 2 \end{cases}$

40. $f(x) = \begin{cases} 1 - x & \text{if } x < -2 \\ 5 & \text{if } x \ge -2 \end{cases}$

41. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

42. $f(x) = \begin{cases} 2x + 3 & \text{if } x < -1 \\ 3 - x & \text{if } x \ge -1 \end{cases}$

43. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ 1 & \text{if } -1 \le x \le 1 \\ -1 & \text{if } x > 1 \end{cases}$

44. $f(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \le x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$

45. $f(x) = \begin{cases} 2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$

46. $f(x) = \begin{cases} 1 - x^2 & \text{if } x \le 2 \\ x & \text{if } x > 2 \end{cases}$

47. $f(x) = \begin{cases} 0 & \text{if } |x| \le 2 \\ 3 & \text{if } |x| > 2 \end{cases}$

48. $f(x) = \begin{cases} x^2 & \text{if } |x| \le 1 \\ 1 & \text{if } |x| > 1 \end{cases}$

49. $f(x) = \begin{cases} 4 & \text{if } x < -2 \\ x^2 & \text{if } -2 \le x \le 2 \\ -x + 6 & \text{if } x > 2 \end{cases}$

50. $f(x) = \begin{cases} -x & \text{if } x \le 0 \\ 9 - x^2 & \text{if } 0 < x \le 3 \\ x - 3 & \text{if } x > 3 \end{cases}$

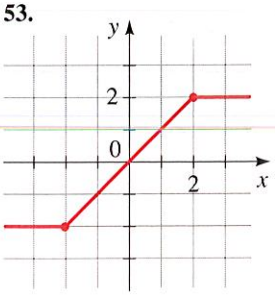
51–52 ■ Use a graphing device to draw the graph of the piecewise defined function. (See the margin note on page 162.)

51. $f(x) = \begin{cases} x + 2 & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$

52. $f(x) = \begin{cases} 2x - x^2 & \text{if } x > 1 \\ (x - 1)^3 & \text{if } x \le 1 \end{cases}$

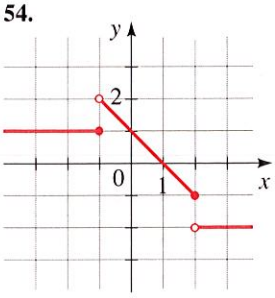
53–54 ■ The graph of a piecewise defined function is given. Find a formula for the function in the indicated form.

53.



$f(x) = \begin{cases} \quad & \text{if } x < -2 \\ \quad & \text{if } -2 \le x \le 2 \\ \quad & \text{if } x > 2 \end{cases}$

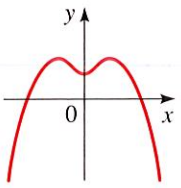
54.



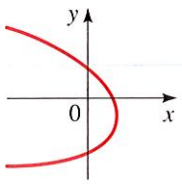
$f(x) = \begin{cases} \quad & \text{if } x \le -1 \\ \quad & \text{if } -1 < x \le 2 \\ \quad & \text{if } x > 2 \end{cases}$

55–56 ■ Determine whether the curve is the graph of a function of x.

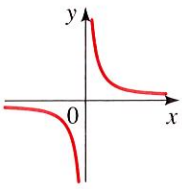
55. (a)



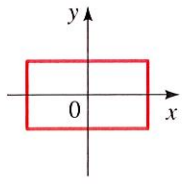
(b)



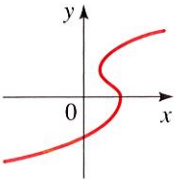
(c)



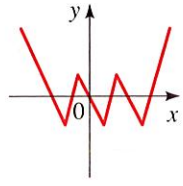
(d)



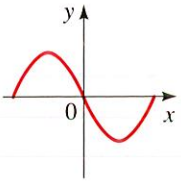
56. (a)



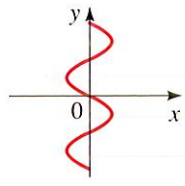
(b)



(c)

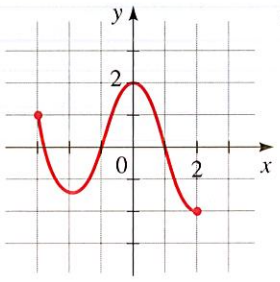


(d)

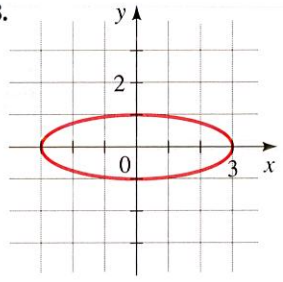


57–60 ■ Determine whether the curve is the graph of a function x. If it is, state the domain and range of the function.

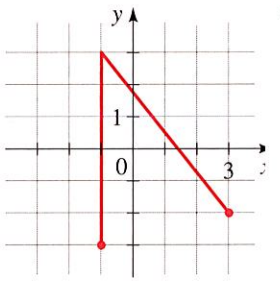
57.



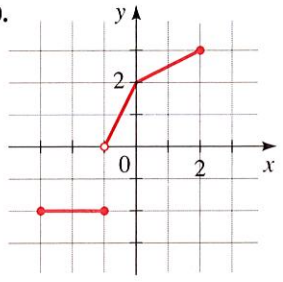
58.



59.



60.



61–72 ■ Determine whether the equation defines y as a function of x. (See Example 10.)

61. $x^2 + 2y = 4$

62. $3x + 7y = 21$

63. $x = y^2$

64. $x^2 + (y - 1)^2 = 4$

65. $x + y^2 = 9$

66. $x^2 + y = 9$

67. $x^2y + y = 1$

68. $\sqrt{x} + y = 12$

69. $2|x| + y = 0$

70. $2x + |y| = 0$

71. $x = y^3$

72. $x = y^4$

73–78 ■ A family of functions is given. In parts (a) and (b) graph all the given members of the family in the viewing rectangle indicated. In part (c) state the conclusions you can make from your graphs.

73. $f(x) = x^2 + c$

(a) $c = 0, 2, 4, 6$; $[-5, 5]$ by $[-10, 10]$

(b) $c = 0, -2, -4, -6$; $[-5, 5]$ by $[-10, 10]$

(c) How does the value of c affect the graph?

74. $f(x) = (x - c)^2$

(a) $c = 0, 1, 2, 3$; $[-5, 5]$ by $[-10, 10]$

(b) $c = 0, -1, -2, -3$; $[-5, 5]$ by $[-10, 10]$

(c) How does the value of c affect the graph?

75. $f(x) = (x - c)^3$

(a) $c = 0, 2, 4, 6$; $[-10, 10]$ by $[-10, 10]$

(b) $c = 0, -2, -4, -6$; $[-10, 10]$ by $[-10, 10]$

(c) How does the value of c affect the graph?

76. $f(x) = cx^2$

(a) $c = 1, \frac{1}{2}, 2, 4$; $[-5, 5]$ by $[-10, 10]$

(b) $c = 1, -1, -\frac{1}{2}, -2$; $[-5, 5]$ by $[-10, 10]$

(c) How does the value of c affect the graph?

77. $f(x) = x^c$

(a) $c = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}$; $[-1, 4]$ by $[-1, 3]$

(b) $c = 1, \frac{1}{3}, \frac{1}{5}$; $[-3, 3]$ by $[-2, 2]$

(c) How does the value of c affect the graph?

78. $f(x) = 1/x^n$

(a) $n = 1, 3$; $[-3, 3]$ by $[-3, 3]$

(b) $n = 2, 4$; $[-3, 3]$ by $[-3, 3]$

(c) How does the value of n affect the graph?

79–82 ■ Find a function whose graph is the given curve.

79. The line segment joining the points $(-2, 1)$ and $(4, -6)$

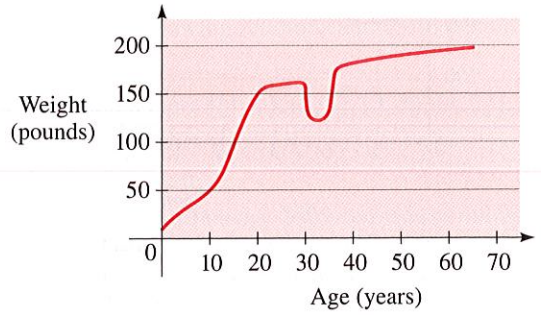
80. The line segment joining the points $(-3, -2)$ and $(6, 3)$

81. The top half of the circle $x^2 + y^2 = 9$

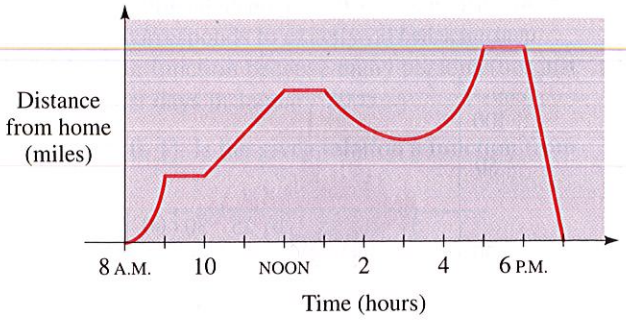
82. The bottom half of the circle $x^2 + y^2 = 9$

Applications

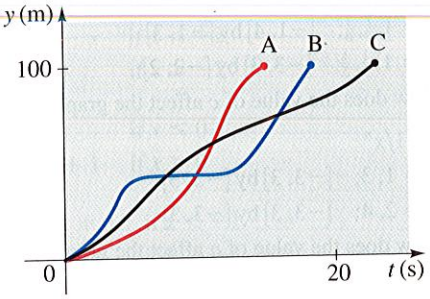
83. Weight Function The graph gives the weight of a certain person as a function of age. Describe in words how this person's weight has varied over time. What do you think happened when this person was 30 years old?



84. Distance Function The graph gives a salesman's distance from his home as a function of time on a certain day. Describe in words what the graph indicates about his travels on this day.



85. Hurdle Race Three runners compete in a 100-meter hurdle race. The graph depicts the distance run as a function of time for each runner. Describe in words what the graph tells you about this race. Who won the race? Did each runner finish the race? What do you think happened to runner B?

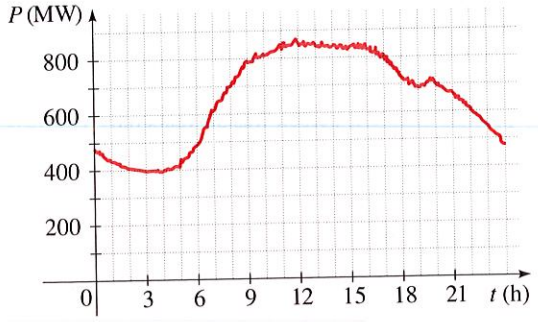


86. Power Consumption The figure shows the power consumption in San Francisco for September 19, 1996 (P is measured in megawatts; t is measured in hours starting at midnight).

(a) What was the power consumption at 6 A.M.? At 6 P.M.?

(b) When was the power consumption the lowest?

(c) When was the power consumption the highest?



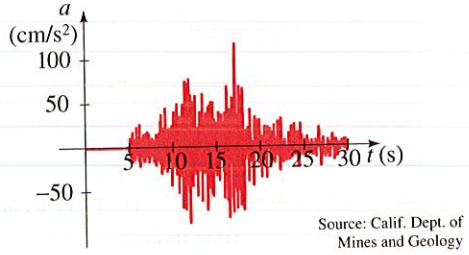
Source: Pacific Gas & Electric

87. Earthquake The graph shows the vertical acceleration of the ground from the 1994 Northridge earthquake in Los Angeles, as measured by a seismograph. (Here t represents the time in seconds.)

(a) At what time t did the earthquake first make noticeable movements of the earth?

(b) At what time t did the earthquake seem to end?

(c) At what time t was the maximum intensity of the earthquake reached?



Source: Calif. Dept. of Mines and Geology

88. Utility Rates Westside Energy charges its electric customers a base rate of $6.00 per month, plus 10¢ per kilowatt-hour (kWh) for the first 300 kWh used and 6¢ per kWh for all usage over 300 kWh. Suppose a customer uses x kWh of electricity in one month.

(a) Express the monthly cost E as a function of x.

(b) Graph the function E for $0 \le x \le 600$.

89. Taxicab Function A taxi company charges $2.00 for the first mile (or part of a mile) and 20 cents for each succeeding tenth of a mile (or part). Express the cost C (in dollars) of a ride as a function of the distance x traveled (in miles) for $0 < x < 2$, and sketch the graph of this function.

90. Postage Rates The domestic postage rate for first-class letters weighing 12 oz or less is 37 cents for the first ounce (or less), plus 23 cents for each additional ounce (or part of an ounce). Express the postage P as a function of the weight x of a letter, with $0 < x \le 12$, and sketch the graph of this function.

Discovery • Discussion

91. When Does a Graph Represent a Function? For every integer n, the graph of the equation $y = x^n$ is the graph of a function, namely $f(x) = x^n$. Explain why the graph of $x = y^2$ is *not* the graph of a function of x. Is the graph of $x = y^3$ the graph of a function of x? If so, of what function of x is it the graph? Determine for what integers n the graph of $x = y^n$ is the graph of a function of x.

92. Step Functions In Example 8 and Exercises 89 and 90 we are given functions whose graphs consist of horizontal line segments. Such functions are often called *step functions*, because their graphs look like stairs. Give some other examples of step functions that arise in everyday life.

93. Stretched Step Functions Sketch graphs of the functions $f(x) = [\![x]\!]$, $g(x) = [\![2x]\!]$, and $h(x) = [\![3x]\!]$ on separate graphs. How are the graphs related? If n is a positive integer, what does the graph of $k(x) = [\![nx]\!]$ look like?

94. Graph of the Absolute Value of a Function

(a) Draw the graphs of the functions $f(x) = x^2 + x - 6$ and $g(x) = |x^2 + x - 6|$. How are the graphs of f and g related?

(b) Draw the graphs of the functions $f(x) = x^4 - 6x^2$ and $g(x) = |x^4 - 6x^2|$. How are the graphs of f and g related?

(c) In general, if $g(x) = |f(x)|$, how are the graphs of f and g related? Draw graphs to illustrate your answer.